Fourier-Based Image Registration Techniques

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1/14/2002       HSS       2
Main Results

• Fast normalized correlation for translations
• High precision phase-correlation for rotation
• Reduced effects of occlusions from clouds
The Goals

• High speed, 10 to 50 ops per pixel
• Improved precision
  – High precision rotation discovery
  – Removes occluded pixels (translation only)
Problem Statement
The Problem

Physical Scene

Image 1

Image 2
Example: Zurich (Landsat)

2/8/90

7/8/92
What has changed?

- Register images (in 2D or 3D)
- Eliminate lighting effects
- Find differences
- Explain differences
  - Clouds
  - Crop cover
  - Catastrophic causes
Fourier-Based Normalized Correlation
Correlations

Define the *correlation* of \( x \) and \( y \), denoted as \( (x \cdot y) \) to be:

\[
(x \cdot y)_j = \sum_{i=0}^{i=N-1} x_i y_{(i+j) \mod N}
\]

Let \( X, Y \) be FFTs of \( x \) and \( y \).

\[
(x \cdot y)_j = \left( \frac{1}{N} \right) \sum_r X_r Y_r W_{jr}
\]

where \( \bar{X} \) denotes complex conjugate of \( X \).
Correlation Computation

\[ \rho(x, y) = \frac{\sum x_i y_i - \left( \frac{1}{N} \right) \sum x_i \sum y_i}{\sqrt{\left( \sum x_i^2 - \left( \frac{1}{N} \right) \left( \sum x_i \right)^2 \right) \left( \sum y_i^2 - \left( \frac{1}{N} \right) \left( \sum y_i \right)^2 \right)}} \]

\[ = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \]
CORRELATION COEFFICIENT

\[ \rho(x, y) = \frac{x \cdot y - \left( \frac{1}{h \cdot m} \right)(x \cdot m)(h \cdot y)}{\sqrt{\left( x^2 \cdot m \right) - \left( \frac{1}{h \cdot m} \right)(x \cdot m)^2} \left( h \cdot y^2 \right) - \left( \frac{1}{h \cdot m} \right)(h \cdot x)^2} \]

where \( m \) is a mask for \( y \) and \( h \) is a mask for \( x \)
Other Criteria

• Intensity differences
• Unnormalized correlations
• Sum of squares of differences
**SPEED UP**

512 x 1024, template = 217 x 231, $6.4 \cdot 10^{11}$ ops

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256 x 256, template = 64 x 64, $8.2 \cdot 10^9$ ops

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Accurate Rotation
Fast Rotation finding

- Theorem: The rotation of the Fourier transform of $f(x)$ is equal to the Fourier transform of the rotation of $f(x)$.

$$FRx = RFx$$

where $F$ is Fourier transform and $R$ is rotation.
Practical Use

Let $f(x) = g(Rx-x_0)$

$$F(\omega) = G(R\omega)e^{-j2\pi\omega x_0}$$

$$|F(\omega)| = |G(R\omega)|$$

Rotate the magnitude of the transform of $G(\omega)$ until it most like the magnitude of $F(\omega)$. 
Phase correlation of $F(\omega), G(\omega)$

Let $f(\rho, \theta)$ be $|F(\omega)|$ in polar coordinates.
Phase correlate $f(\rho, \theta)$ to $g(\rho, \theta)$, vs. $\theta$
Let $F$ be Fourier transform of $f(\rho, \theta)$.

$$\frac{F(\rho, \theta) \hat{G}(\rho, \theta)}{|F(\rho, \theta)| |G(\rho, \theta)|} = \frac{F(\rho, \theta) \hat{F}(\rho, \theta) e^{-2j\pi\theta_0}}{|F(\rho, \theta)| |F(\rho, \theta)|} = e^{-2j\pi\theta_0}$$

Inverse transform is a delta function at $\theta_0$
What you actually get

Why is there a false peak?
Why is height of the correct peak so small?

From Lucchese et al.
Explanation

- Fundamental problem:
  \[ FR_x \neq RF_x \]
  for finite sampled images

- Rotationally dependent aliasing causes false peaks and lowers peak heights at correct locations.
Finite-Transform Pairs

\[ x \quad FRx \quad RFx \]
Finite-Transform Pairs

\[ x \]

\[ FRx \]

\[ RFx \]
The Artifacts of Aliasing
Enhanced Phase Correlation
Reducing Aliasing artifacts

- Window the image to eliminate boundary artifacts
- Remove central frequencies -- these contribute most to rotational aliasing
- Phase correlate over 180°, not over 360°
Blackman Window

Windowing reduces effects at the rectangular boundaries

Multiply the image point-by-point by the window function
Example: Images and Transforms

Raw
Windowed
HSS
Disk Only
Results: No Window

Small and Large Low-Pass Cutoffs

Low-Pass Cutoff

Correct Peaks
Results: Windowing

Small and Large Low-Pass Cutoffs

Correct Peaks S/N > 110
Higher Peak

- Phase correlation normalizes all frequency coefficients to unit length, including $F(0,0)$.
- Sum of magnitudes must equal 1.
- False peaks reduce height of correct peak.
- $360^\circ$ phase correlations have two cycles, so no peak can exceed 0.50.
- Remove false peaks, and correlate over $180^\circ$ to increase peak height.
Summary

• Fast normalized correlations, with occlusion masks
• Handles 10 images/second in Matlab, images of size 512 by 512
• Ideal for searching moderate to large areas
• Improved precision of fast phase correlation