The Semi-Analytic Mode Matching (SAMM) Algorithm for Efficient Computation of Nearfield Scattering in Lossy Ground from Borehole Sources

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Abstract

The three-dimensional semi-analytic mode matching (SAMM) algorithm is used to determine nearfield scattering from underground targets in lossy soil, where the source is a dipole placed within a borehole in the ground. Scattering is described by moderately low-order supersolutions of spherical modes placed at multiple user-specified coordinate scatterer locations (CSCs); the mode coefficients are found numerically by least-squares fitting all boundary conditions at discrete points along the relevant interfaces while at the same time obeying radiation conditions. SAMM results are compared with a completely different method: the Half-Space Born Approximation (HSBA). Good agreement between methods validates both algorithms. Unlike HSBA, SAMM is not a perturbative algorithm and does not require small dielectric or volumetric perturbations of the half-space geometry. In general, SAMM is a faster and more robust approach for inverse problems, key scattering features can often be determined with low-order modes, sacrificing details for speed.

Value Added to CenSSIS

SAMM Algorithm: Basic Principles

- Identify 2D (cylindrical) / 3D (spherical) modes which satisfy wave equations and radiation conditions. The appropriate (spherical) Bessel functions are chosen so that fields remain finite away from sources and radiation conditions hold in the far field.
- Place points on the surface of all interfaces (e.g., air/ground and target) for which boundary conditions on all three (cylindrical) axis Cartesian field components are to be matched, with point spacing and number chosen to maximize algorithm speed and minimize computational storage.
- Choose locations and orientations of coordinate scatterer centers (CSCs) from which the waves will emerge with as few undetermined coefficients as possible.
- Construct a dense matrix $F$ linking the undetermined mode coefficients ($e$) with the field distribution ($h$) at each point on the material interfaces.
- Minimize the matrix equation $F e = b$ using singular value decomposition, selecting the (small) group of mode coefficients which best fit the (large) number of boundary conditions; the size of built-in error function $F e = b$ indicates how successfully SAMM has modeled scattering.

The SAMM solution will be optimal in the nearfield region where it is properly constrained and inaccurate in the extrapolated region away from the fitting points.

SAMM Equations

The scalar Debye potential is a solution of the Helmholtz equation and forms the starting point for the SAMM algorithm, with 3D solutions to the Debye potential given by spherical modes:

$$\Phi(r, \theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_n^m J_n(kr) P_n^m(\cos \theta),$$

where $J_n(kr)$ and $P_n^m(\cos \theta)$ are spherical Bessel and associated Legendre polynomials, respectively.

Electric and magnetic fields, given by differential operations on the Debye potential, are thus also supersolutions of spherical modes. The SAMM solution is a spherical Bessel or spherical Hankel function of the first kind, and $\Phi(r, \theta, \phi)$ is an associated Legendre polynomial. The coordinates $r$ and $\phi$ are in spherical coordinates, but $\theta$ is in the plane of the incident wavefront.

As well as having a spatial location, each CSC may also have a user-specified orientation. The series expansion is truncated by choosing $N$ and $M$, the maximum radial and angular mode indices, such that $r < N$ and $\theta < M$ in the same region of the half-space.

Half-Space Born Approximation (HSBA)

The 3D half-space Born approximation (HSBA) algorithm uses plane wave decomposition techniques to construct spherical waves from the superposition of all possible plane wave modes in half-space geometries containing well-characterized dispersive media, where fields are found from the vector Helmholtz equation using the dyadic Green’s function. Classical Fresnel theory is used in the resulting plane waves at the plane interface, and the first-order Born approximation (weak scatterer assumption) is applied to establish a linear relationship between the field and the object characteristic function. SAMM and HSBA are two very different methods for generating fields from pre-specified geometries but should yield similar results for identical geometries and can be used for cross-validation.

Choosing CSCs for Underground Target with Borehole Dipole Source

Step 1 – No Target: Use purple CSCs (no need for azimuthal modes) to generate fields for buried dipole source configuration with no target

Step 2 – Add Target: Use black CSCs within target and its image to generate total scattered fields, using Step 1 fields as the source

Comparison of SAMM and HSBA

- SAMM solution is optimal in the nearfield region of the half-space.
- SAMM solution will be optimal in the nearfield region where it is properly constrained and inaccurate in the extrapolated region away from the fitting points.

3D DNA Plotted Prolate Spheroid

Buried in Wet Sand with a Dipole Source

- Frequency: 1.4 GHz, Grid Step Size $\lambda/20 = 0.0047$ m
- 54-dipole source located at $(-17c, 17c, -10b)$
- Cartesian field components determined by 5 image CSCs located within the air
- Field mismatch ($\Phi - \Phi^\text{in}$) reduced to a point source at $(-3c, 3c, -10b)$

SAMM (Matlab laptop) takes about 1/10 the computational time of HSBA (Alpha workstation)

References