Superresolution of Buried Objects in Layered Media by Near-Field Electromagnetic Imaging

Sean K. Lehman
Lawrence Livermore National Laboratory

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Introduction

- Definition of Problem Scenario
- Development of Forward Model (Porter-Bojarski Equation)
- The Porter-Bojarski Equation Contains No Evanescent Field
- Total Field Model
- Planar Forward Model
- Inverse
- Proof-of-Principle
- FDTD Example of Two Aluminum Poles
- LLNL MIR Scan of Two Aluminum Poles
- LLNL MIR Scan of Aluminum Resolution Phantom
- Conclusions
(a) We want to determine non-invasively the location and nature of objects buried in a layered medium and residing within the near-field of the radiation. (b) Two dimensional simplification. A scattering object \( o(r, t) \) is buried in a medium. A transmitter/receiver launches a pulse \( p(r, t) \) into the medium and records the echo from the scatterer.

Primary source launches a pulse
\[
\left[ \nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right] u_c(r, t) = -p(r, t)
\]

Secondary source re-radiates the pulse
\[
\left[ \nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right] u_s(r, t) = -q(r, t)
\]
Incident & Scattered Fields

Fourier transform in time

\[
\begin{align*}
\left(\nabla^2 + k_0^2(\omega)\right) u_i(r, \omega) &= -p(r, \omega) \\
\left(\nabla^2 + k_0^2(\omega)\right) u_s(r, \omega) &= -q(r, \omega)
\end{align*}
\]

\[
\begin{align*}
 u_i(r, \omega) &= \int_{V_m} d\mathbf{r'} p(\mathbf{r'}, \omega) G(\mathbf{r} - \mathbf{r'}, \omega) \\
u_s(r, \omega) &= \int_{V_m} d\mathbf{r'} q(\mathbf{r'}, \omega) G(\mathbf{r} - \mathbf{r'}, \omega) \\
q(r, \omega) &= k_0^2(\omega) \sigma(r, \omega) \tau(r, \omega) \\
\sigma(r, \omega) &\in V_o
\end{align*}
\]

Total field

\[
u(r, \omega) = u_i(r, \omega) + u_s(r, \omega)
\]

Development of Model

Follow Porter & Bojarski:

- Define a “generalized” time-reversed field within the measurement volume;
- Derive the relationship between the measured scattered field and the unknown object distribution;
- The Porter-Bojarski model does not contain evanescent field;
- Develop new forward model;
- Develop new “extended resolution” inversion algorithm.
The Porter-Bojarski Equation — I

Analyze this scenario using the Porter-Bojarski *generalized field* inside the measurement volume:

\[
\chi(\mathbf{r}, \omega) \equiv \int_{S_M} dS' \left[ G^*(\mathbf{r} - \mathbf{r}', \omega) \partial_{r'} u(\mathbf{r}', \omega) - u(\mathbf{r}', \omega) \partial_{r'} G^*(\mathbf{r} - \mathbf{r}', \omega) \right]
\]

Apply Green’s theorem,

\[
\chi(\mathbf{r}, \omega) = u(\mathbf{r}, \omega) - \int_{V_M} d\mathbf{r}' \, G^*(\mathbf{r} - \mathbf{r}', \omega) q(\mathbf{r}', \omega),
\]

and substitute the expression for the *non-time-reversed total field*,

\[
\chi(\mathbf{r}, \omega) = u_i(\mathbf{r}, \omega) + \int_{V_M} d\mathbf{r}' \, G(\mathbf{r} - \mathbf{r}', \omega) q(\mathbf{r}', \omega) - \int_{V_M} d\mathbf{r}' \, G^*(\mathbf{r} - \mathbf{r}', \omega) q(\mathbf{r}', \omega),
\]

\[
= u_i(\mathbf{r}, \omega) + 2i \int_{V_M} d\mathbf{r}' \, G_{im}(\mathbf{r} - \mathbf{r}', \omega) q(\mathbf{r}', \omega)
\]

The Porter-Bojarski Equation — II

Move the incident field to the left hand side to obtain the *generalized scattered field*:

\[
\chi(\mathbf{r}, \omega) - u_i(\mathbf{r}, \omega) = 2i \int_{V_M} d\mathbf{r}' \, G_{im}(\mathbf{r} - \mathbf{r}', \omega) q(\mathbf{r}', \omega)
\]

The left hand side reduces to the scattered field on the measurement surface:

\[
\int_{S_M} dS' \left[ G^*(\mathbf{r} - \mathbf{r}', \omega) \partial_{r'} u(\mathbf{r}', \omega) - u(\mathbf{r}', \omega) \partial_{r'} G^*(\mathbf{r} - \mathbf{r}', \omega) \right] = 2i \int_{V_M} d\mathbf{r}' \, G_{im}(\mathbf{r} - \mathbf{r}', \omega) q(\mathbf{r}', \omega)
\]

This is the Porter-Bojarski equation
\[ [\nabla^2 + k_0^2(\omega)] G(\mathbf{r} - \mathbf{r}', \omega) = -\delta(\mathbf{r} - \mathbf{r}') \]

<table>
<thead>
<tr>
<th>Real Part</th>
<th>Imaginary Part (Propagating Field)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ [\nabla^2 + k_0^2(\omega)] G_{re} = -\delta(\mathbf{r} - \mathbf{r}') ]</td>
<td>[ [\nabla^2 + k_0^2(\omega)] G_{im} = 0 ]</td>
</tr>
</tbody>
</table>

Fourier transform

\[
\tilde{G}_{re} = \frac{1}{|\mathbf{k}|^2 - k_0^2(\omega)} \quad \tilde{G}_{im} = \frac{\pi}{2k_0(\omega)} \delta(|\mathbf{k}| - k_0(\omega))
\]

**The Porter-Bojarski Equation Spectrum**

Fourier transform in space

\[
\tilde{G}_{im}(\mathbf{k}, \omega) \left[ [|\mathbf{k}|^2 - k_0^2(\omega)] U_m(\mathbf{k}, \omega) \right] = 2 \tilde{G}_{im}(\mathbf{k}, \omega) \tilde{Q}(\mathbf{k}, \omega)
\]

Recall:

\[
\tilde{G}_{im}(\mathbf{k}, \omega) = \frac{\pi}{2k_0(\omega)} \delta(|\mathbf{k}| - k_0(\omega))
\]

The Porter-Bojarski equation contains only propagating fields.

Thus, another forward model must be developed. One which contains the evanescent field.
Use the Porter-Bojarski time-reversed field model but do not replace the total field by the non-time-reversed field.

\[
\mathbf{u} (\mathbf{r}, \omega) = \int_{V_{sr}} d\mathbf{r}' \, G^* (\mathbf{r} - \mathbf{r}', \omega) \mathbf{q} (\mathbf{r}', \omega) + \int_{S_{sr}} dS' \left[ G^* (\mathbf{r} - \mathbf{r}', \omega) \partial_{\mathbf{n}} \mathbf{u} (\mathbf{r}', \omega) - \mathbf{u} (\mathbf{r}', \omega) \partial_{\mathbf{n}} G^* (\mathbf{r} - \mathbf{r}', \omega) \right]
\]

Apply Green’s theorem

\[
\begin{align*}
\mathbf{u}_e (\mathbf{r}, \omega) = & \int_{S_{sr}} dS' \left[ G^* (\mathbf{r} - \mathbf{r}', \omega) \partial_{\mathbf{n}} \mathbf{u}_e (\mathbf{r}', \omega) - \mathbf{u}_e (\mathbf{r}', \omega) \partial_{\mathbf{n}} G^* (\mathbf{r} - \mathbf{r}', \omega) \right] \\
= & \int_{V_{sr}} d\mathbf{r}' \, G^* (\mathbf{r} - \mathbf{r}', \omega) \mathbf{q} (\mathbf{r}', \omega)
\end{align*}
\]

\[
U_s (\mathbf{k}, \omega) - \tilde{G}_{nm} (\mathbf{k}, \omega) \left[ |\mathbf{k}|^2 - k_0^2 (\omega) \right] U_s (\mathbf{k}, \omega) = \tilde{G} (\mathbf{k}, \omega) \tilde{Q} (\mathbf{k}, \omega)
\]

Contains Evanescent Field

Contains Evanescent Field
Planar Model

Distort the measurement surface, $S_M$, such that it becomes a hemisphere extending to infinity over the $z = z_M$ plane.
where $S(k_0(\omega) - |k_\perp|)$ is the unit step function, and

$$\gamma(k_\perp, \omega) \equiv \sqrt{k_0^2(\omega) - |k_\perp|}$$

This represents a mapping of the secondary source spectrum evaluated at the locus of points given by $(k_\perp, \pm\gamma^*(k_\perp, \omega))$ in a generalized Fourier space, to the planar spectrum of the measured field.
Two-Dimensional Multimonostatic Expression

Forward planar model

\[
\tilde{E}_x^P(k_x, z_M, \omega) \left[1 \pm \frac{1}{2} S(k_0(\omega) \pm |k_\omega|)\right] =
\frac{-i A(\omega) \ k_0^2(\omega) e^{\pm \gamma^* (k_x \omega) \pm \pi x}}{2\gamma^* (k_x, \omega)} \tilde{O}(k_x, -k_0(\omega) \pm \gamma^* (k_x, \omega)).
\]

Theoretical inverse

\[
\alpha_{x,z}(x, z) = \frac{2}{(2\pi)^2 i_{0}} \int_{0}^{\infty} d\omega \ e^{i k_0(\omega) z} \int_{0}^{\infty} dk_x \ e^{S(k_0(\omega) - |k_x|)} \times
\left[-\gamma^* (k_x, \omega) \pm k_0(\omega) \right] \left[1 \pm 0.5 S(k_0(\omega) - |k_\omega|)\right] \times
\tilde{E}_x(k_x, z_M, \omega) e^{-i k_x z} e^{\pm \pi x |k_x \omega| (z - i \pi x)}
\]
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\[ o_{\tau r}(x, z) = \frac{2}{(2\pi)^2 v_0} \int_0^\infty \omega d\omega \ \frac{e^{ih_0(\omega)z}}{k_0^2(\omega)A(\omega)} \int dk_{\tau r} \ iS(k_\tau(\omega) - |k_\tau|) \times \]
\[ | -\gamma^*(k_{\tau r}, \omega) \pm h_0(\omega) | [1 \pm 0.5 S(k_0(\omega) - |k_\tau|)] \times \]
\[ \tilde{U}_k(k_{\tau r}, M, \omega) \ M(k_{\tau r}, z, \omega) \ e^{-ik_{\tau r}x} \ e^{i\pi \gamma^*(k_{\tau r}, \omega)(z-zA)} \]

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- \( M(k_{\tau r}, z, \omega) \) is a cutoff window which limits the extent of the evanescent field information used in the reconstruction;
- It limits the exponential expression \( e^{\gamma^*(k_{\tau r}, \omega)z} \) which diverges in the evanescent region;
- It dictates the wavenumber limit, \( k_{\tau r} \), beyond which no object spectral information is contained in the measured planar field spectrum.

\[ k_{\tau r}(z) = \sqrt{\left( \frac{1}{z \ln \tau} \right)^2 + k_0^2(\omega)}, \]

where \( \tau \) is a selected decay below which there is no signal.

\[ M(k_{\tau r}, z, \omega) \equiv S(k_{\tau r}(z) - |k_\tau|) \]
Evanescent Cutoff Window

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Proof-of-Principle

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FDTD Example

FDTD Simulation — MIR Pulse & DOG Model

\[ p(t) = -\left(\sigma e^{1/2}\right) \frac{t-t_0}{\sigma^2} e^{-\frac{(t-t_0)^2}{2\sigma^2}} \]
Each aluminum cylindrical scatterer is 1.5 cm in diameter; 
The transmitter/receiver line is 11 cm above the surface (red line); 
The phantom is 3 cm below the surface; 
The material has a relative permittivity of 3 and a conductivity of 0.1 Siemens/meter (dry sand).
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**FDTD Simulation — Reconstructions**

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**LLNL MIR Scan of Two Aluminum Poles**
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**Backscattered Field**

\[ u(x,z_0,t) \]

\( x \) (meters)

\( t \) (ns)

0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6

0 1 2 3 4 5 6

(Background removed)

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**Reconstructions**

Abs(\( o_{lp}(x,z) \))

Abs(\( o_{xr}(x,z) \))

\( x \) (meters)

\( z \) (meters)

-0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4

0 0.1 0.2 0.3 0.4

-1 -0.5 0 0.5 1

0 0.2 0.4 0.6 0.8 1

0.2 0.4 0.6
Each aluminum cylindrical scatterer is 1.5 cm in diameter
Each aluminum cylindrical scatterer is 1.5 cm in diameter;
The transmitter/receiver line is 11 cm above the surface (red line).
The phantom is 3 cm below the surface;
Buried Resolution Phantom Reconstructions

Conclusions

- The most popular DT forward model does not include evanescent field information;
- We developed a new model which contains the evanescent field;
- The spectral mapping of the secondary source spectrum exists in a generalized Fourier space;
- We derived an “extended resolution” planar inversion algorithm under the Born approximation;
- Reconstructions on aluminum targets are very good considering the scattering violates the Born approximation.