Shape Simulations and Image Segmentation for image-guided radiotherapy

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1. Introduction to Shape Simulation

- Sampling from distributions \( r(x) \) is important in many fields.
- We introduce a method of sampling shapes from an arbitrary distribution \( r(x) \) defined over the space of simple closed curves.
- In radiotherapy, a single treatment plan is not accurate due to organ's deformation. Another approach is to define a probability distribution over possible shapes of the target organ, sample shapes from it, precompute corresponding treatment plans, and select the best matching plan on the day of treatment.
- State of the art:
  - Markov chain Monte Carlo simulation and Metropolis-Hastings algorithm.
  - Fan et al. proposed a similar algorithm but their method can't be extended to 3D problem. Further more, the computational complexity of their method is high.

Our proposed algorithm:

- The contour \( C \) is embedded as the zero level set of a signed distance function \( \phi \).
- The proposal strategy is to choose a source point \( S \) and locally "push" the curve in the normal direction at \( S^{-} \):
  \[ \phi = \phi^{-} + \frac{1}{\epsilon} \nabla \phi \left( S^{-} \right) \cdot \mathbf{n} \]
  where \( D \) is the geodesic distance and \( F(x) \) is the footpoint of \( x \).
- We can show that the proposal distribution is:
  \[ \phi(\phi^{-} + \frac{1}{\epsilon} \nabla \phi \left( S^{-} \right) \cdot \mathbf{n}) = \phi \left( S^{-} \right) \]

Figure 1: Perturbation examples. The black and green curves are \( C^{-} \) and \( C \), the blue square is the source point.

The overall proposed Markov Chain Monte Carlo curve sampling algorithm:

Initialize \( \phi \)

for \( i = 1 \) to \( N \) do
  Sample \( u \sim U(0, 1) \)
  Randomly pick \( \epsilon^{-1} \) over the zero level set of \( \phi^{-} \)
  Generate \( \phi^+ \)
  if \( u < \min \left( \frac{\phi(\phi^+)}{\phi(\phi^{-})} \right) \) then
    \( \phi^{-} = \phi^+ \)
  end if
end for

2. Introduction to Level Set Segmentation

- State of the art:
  - In medical images, objects often exhibit inhomogeneous intensity distributions that are problematic for segmentation methods that use no prior information learned from training data.
  - Existing approaches often need a heuristic weighting parameter to balance the contribution of image energy and shape energy.
- Our approach:
  - Uses global descriptors for both image and the shape, therefore no balance among image energy and shape energy.
  - When training images are available, our method can learn the intensity distributions with nonparametric density estimation techniques and drive the segmentation accordingly.

Level Set Segmentation as Bayesian Inference

Given training shapes \( \phi \) and training images \( \tilde{I} \), the segmentation problem is formulated as:

- The contour \( C \) is represented by the zero level set of a signed distance function (SDF) \( \phi \).
- The segmentation is achieved by minimizing a energy cost functional \( E(\phi) \) using the calculus of variations technique.

\[ \min_{\phi} E(\phi) \]
where \( E(\phi) = \int_0^{2\pi} \frac{1}{2} \rho_d(\phi) \| \phi \|_2^2 + \frac{1}{2} \rho_s(\phi) \| \phi \|_2^2 - \frac{1}{2} \rho_n(\phi) \| \phi \|_2^2 \]

We characterize the image information using the histogram of the area inside and outside the evolving contour \( \phi(\phi^{-}) \).

Results of Level Set Image Segmentation

(a) (b) (c) (d) Data set 1–3

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