Sequential Sparsification for Change Detection
Necmiye Ozay, Mario Sznaier and Octavia I. Camps
Dept. of Electrical and Computer Engineering

**GOAL:** Segmenting a vector valued sequence into an unknown number of sub-sequences, each represented by an affine parametric model.

**APPLICATIONS:**
- Video segmentation: Fit affine subspaces
- Dynamic Texture segmentation: Fit hybrid ARX models

**EXISTING APPROACHES:**
- Hybrid/Mixture Models for Clustering:
  - Don't take into account data ordering
  - Might require number of clusters
  - Not always robust to noise
- Shot Detection Algorithms:
  - Too many parameters to tune
  - Not suitable to detect changes in dynamics

**KEY OBSERVATION:** Sparsity
Segmenting a sequence \(\{x(k)\}_{k=0}^{T} \) generated by an unknown model of the form:
\[
H: f\left(p_{\sigma(t)}, \{x(t)\}_{k=t-1}^{t+j}\right) = A(x)p_{\sigma(t)} + b(x) = 0
\]
is equivalent to finding the **switches** of the model.

\[
p_{\sigma(t)} \quad \rightarrow \quad p(t)
\]

\[
g(t) = p(t) - p(t-1)
\]

**SWITCHES** are indicated by \(g(t) \neq 0\)
If the data has \(k<<T\) segments, there are only \(k-1\) switches and \(g(t)\) is **sparse**.

**OPTIMIZATION PROBLEM:**
**MINIMUM NUMBER of SWITCHES**

\[
\min_{p(t), \eta(t)} \|\{g\}\|_{\infty}
\]
subject to
\[
f\left(p(t), \{x(k)\}_{k=t-1}^{t+j}\right) = \eta(t) \quad \forall t
\]
\[
\|\{\eta\}\|_{*} \leq \epsilon
\]

**CONVEX RE-WEIGHTED II NORM RELAXATION** (OFTEN EXACT)

\[
\min_{\zeta, g, p, \eta} \sum_{t=1}^{T-1} w_{t}^{(n)} \zeta_{t}
\]
subject to
\[
\|g(t)\|_{\infty} \leq \zeta_{t} \quad \forall t
\]
\[
f\left(p(t), \{x(k)\}_{k=t-1}^{t+j}\right) = \eta(t) \quad \forall t
\]
\[
\|\{\eta\}\|_{*} \leq \epsilon
\]

**with**
\[
w_{t}^{(n)} = (z_{t}^{(n)} + \delta)^{-1}
\]
and \(z^{(0)} = [11 \ldots 1]^{T}\)

**DYNAMIC TEXTURE SEGMENTATION**
Main motion changes direction halfway.

**CONCLUSIONS:**
Sequential sparsification change detection was solved as a convex problem. The approach is robust to noise and only required a single parameter.

**PUBLICATIONS:**