Validating Four-zeros 2D FDTD Model
Maryam Jalalinia, Carey M. Rappaport
mjalali@ece.neu.edu, rappapor@ece.neu.edu

Abstract:
In this work we model wave propagation in two-dimensional dispersive medium using FDTD (Finite Difference Time Domain) method. For a lossy, dispersive medium, it is significant to model the wave velocity and attenuation over a wide RF bandwidth with a simple and efficient model. Using a four-zeros simple-fractional rational function of the Z-transform variable \( Z = \exp(i\omega t) \) to model conductivity with constant real dielectric constant, we generate a discretized time domain equation which matches measured values over more than two decades of frequency. The simulations show that there is a good agreement between measured and modeled propagation constant and attenuation rate.

State of The Art:
Other dispersive media models: Frequency domain dispersive complex dielectric constant with rational functions of \( j\omega \): Debye and Lorentz models [1,2]. Accurate forward model that can give insight on how to successfully design detection systems.

Significance:
Accurate forward model that can give insight on how to successfully design detection systems.

Propagation in Dispersive Media in Time-Domain:
Z-transformed current:
\[
\mathcal{T}(Z) = \sigma(Z) \mathcal{E}(Z)
\]
Four-Zero Conductivity Rational Function:
\[
\sigma(Z) = Z^4 b_4 Z^{-1} + b_3 Z^{-2} + b_2 Z^{-3} + b_1 Z^{-4} + b_0
\]
The transformed difference equation, with average current value between adjacent steps is:
\[
\mathcal{T}^{-1} F [\mathcal{T}(Z)] \Delta Z b_4 Z^{-1} + b_3 Z^{-2} + b_2 Z^{-3} + b_1 Z^{-4} + b_0
\]
Dispersive FDTD form of Ampere’s law:
\[
\nabla \times \mathbf{E} = \frac{1}{\epsilon_0} \frac{\partial \mathbf{B}}{\partial t} + \sigma \mathbf{E}
\]
The model is implemented by fitting Re(\( \sigma \)) to measured conductivity and Im(\( \sigma \)) to measured real dielectric constant. An initial guess for all allows a simultaneous solution for other parameters. The conductivity and dielectric constant at three representative frequencies for the measured data and the model are equated, and further simple optimization is performed by trial and error. The choice of \( \Delta Z \) defines the model, and determines the usable frequency range.

Modeling Example:
Modeled electric properties and complex wave-number of breast fat tissue vs. data are plotted in following figures which show good agreement.

Conclusions and Future Work:
Four-zeros Z-transform conductivity model was developed as a simple, effective, and accurate way to model dispersion. As an example, breast fat tissue was modeled and compared to data. Stability condition was studied with an example. At last, propagation of wave in air in a 2-D dispersive medium was simulated and results were shown versus time.

References: